References

[1] Zakkay, V. and Krause, E., "Boundary conditions at the outer edge of the boundary layer on blunted conical bodies," Aeronaut. Res. Labs. 62-386 (July 1962).

[2] Klaimon, J. H., "Bow shock correlation for slightly blunted cones," AIAA J. 1, 490–491 (1963).

Linear Programming for Life Support Optimization

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Nomenclature

b_i = ith life support requirement, lb/man-day
x_j = installed weight of the jth life support process, lb
a_{ji} = pounds per day input or output of the ith material, produced by one installed pound of the jth process
W = total weight chargeable to life support system, including effect on radiator weight, lb
c_j = total weight chargeable to jth process, including effect on radiator weight, pounds per installed pound of the jth process

Introduction

LINEAR programming is used to optimize the design and operation of a chemical plant operating with different raw materials, intermediate materials, products, and forms of energy.[1] In this paper linear programming is used to optimize the air, water, and thermal control for a one man-day life support system based on the storage of supplies and wastes. A similar approach could be used to minimize cost or optimize for some other desirable condition for the various types of environmental control systems.

Analysis

The requirements for thermal, atmosphere, and water management for one man-day are listed in Table 1. It is assumed that water vapor is condensed and re-used, and the daily water requirement is reduced accordingly.

Different processes can be selected to provide for the requirements in Table 1. The processes considered in this paper are listed in Table 2, along with the coefficients relating each process with the environmental requirements [Eq. (1)] and with the weight W that is to be minimized [Eq. (2)]:

$$b_i = \sum_j a_{ji}x_j \quad (1)$$

$$W = \sum_j c_j x_j \quad (2)$$

The values shown are presented as typical only and will vary with the exact formulation of each process and the equipment and operating techniques used. The relations are assumed linear as an approximation. Since some of the processes release or absorb more than one material, there are various cross terms. The coefficient that shows the effect of KO_2 on the water balance is negative, $a_{5,2} = -0.16$, because water is absorbed when KO_2 absorbs CO_2 and releases O_2. The coefficient c_j (defined in Table 2) accounts for the required change in radiator weight due to a unit change in the weight of process x_j. For instance, liquid oxygen absorbs heat and reduces radiator weight; therefore, $c_1 = 0.92$. Hydrogen peroxide, on the other hand, releases heat, and $c_2 = 1.59$.

Where there are two more variables than constraints, a plot of the problem can be made on a two-dimensional graph,

Table 1 Life support requirements, lb/man-day

Substance	Quantity	Symbol
Oxygen input	2	b_1
Water input	2.8 (net)	b_2
Carbon dioxide output	2.3	b_3

Table 2 Programming data

Agent	Programming variable	Pertinent coefficients
$O_2(1)$	x_1	$a_{1,1} = 0.8$; $c_1 = 0.92$
H_2O_2	x_2	$a_{2,1} = 0.36$; $a_{2,2} = 0.5$; $c_2 = 1.59$
H_2O	x_3	$a_{3,2} = 0.9$; $c_3 = 1.0$
LiOH	x_4	$a_{4,3} = 0.8$; $a_{4,2} = 0.33$; $c_4 = 1.57$
KO_2	x_5	$a_{5,1} = 0.29$; $a_{5,2} = -0.16$; $a_{5,3} = 0.27$; $c_5 = 1.34$

and the optimum can be found by inspection (see Fig. 1). The lines are determined by solving for the weight in terms of x_2 and the other variables, individually, in succession. The other variable is then set equal to zero, and the various straight lines can be plotted.

Thus, in Fig. 1, the vertical axis represents the total weight, and the horizontal axis represents the weight of hydrogen peroxide and hydrogen peroxide storage equipment. Along the vertical axis the amount of hydrogen peroxide is zero. The various straight lines on the figure represent points where the weight of the process noted is zero. On one side of these lines the weight would be negative, and these sides are excluded as shown by the hatching.

The only allowed solutions are those represented by points within the shaded area. Since the vertical axis represents the weight that is to be minimized, the optimum is represented by the lowest point within the shaded area. This point is on the intersection of the lines where hydrogen peroxide and potassium superoxide are both zero. The nonzero substances are water, liquid oxygen, and lithium hydroxide. These substances would be used in the life support system. The total weight would be a little under 9 lb.

The more general method of solving linear programming problems involves the use of "tableaux." Only a sketch of the method can be given here (see Ref. 2 for details). The basic idea is the same as in the graphical solution. Since the relations are linear, the allowed solutions will represent a convex set in a space with dimensions equal to the variables minus the constraints. This means that there are no "re-entrant ridges," and, if water were placed in a vessel

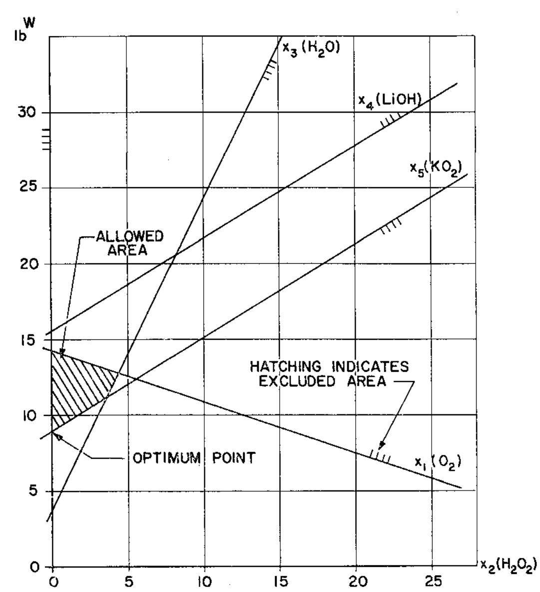

Fig. 1 Graphical solution of linear programming.

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Table 3 Initial tableau

			x_3	x_5
			1.0	1.34
0.92	x_1	0.84	−0.81	0.61
1.59	x_2	3.7	1.8*	−0.54
1.57	x_4	2.9		0.34
		11.2	1.12	−1.11

$$x_1 - 0.81x_3 + 0.61x_5 = 0.84$$
$$x_2 + 1.8x_3 - 0.54x_5 = 3.7$$
$$x_4 + .34x_5 = 2.9$$

Table 4 Final tableau

			x_2	x_5
			1.59	1.34
0.92	x_1	2.5	0.45	0.36
1.0	x_3	2.1	0.56	−0.3
1.57	x_4	2.9		0.34
		8.9	−0.62	−0.78

$$x_1 + 0.45x_2 + 0.36x_5 = 2.5$$
$$x_3 + 0.56x_2 - 0.3x_5 = 2.1$$
$$x_4 + 0.34x_5 = 2.9$$

matching the boundaries of the set, it would always run to the lowest point. This method is not limited to the case where there are two more variables than constraints. The approach is to specify one corner of the convex set by setting a sufficient number of the variables to zero. In the formulation of Vajda,[2] these are termed nonbasic variables. The number of these nonbasic variables equals the total number of variables minus the number of constraints. Having set the assumed nonbasic variables to zero, which specifies one corner of the convex set, the changes of the variable to be minimized (or maximized) are found along the edges of the convex set which lead from the corner of the set that was chosen. Then, a move is made along the edge where the change in the dependent variable is largest in the direction desired. This process is repeated until the changes in the dependent variable along the edges are all opposite to the direction desired. The corner at which this occurs is the optimum.

How this optimizing process can be accomplished through the use of tableaux is illustrated in Tables 3 and 4. For comparison, the problem is the same one that was solved graphically in Fig. 1. The independent variables that are to be set to zero for the first try are selected arbitrarily, and these are listed at the heads of the vertical columns. The other variables are placed next to the left end of the rows in the tableau. The equations of constraint are then solved to give each of the nonzero variables in terms of the zero variables (listed at the top of the columns). The pertinent equations are listed below the tableaux.

The body of each tableau is made up of a column of numbers just to the right of the column of nonzero variables (basic variables) that are adjacent to the extreme left of the tableau. These numbers are the constant terms in the equations listed below the tableau. The row of numbers just below the row of zero (nonbasic) variables that are listed at the top of the tableau are the coefficients c_j that give the effect on weight corresponding to the nonbasic variable listed above. Similar coefficients are listed to the left of the basic variables. Where coefficients or terms are zero they are not entered into the tableau. In the body of the tableau are numbers corresponding to a pair of variables consisting of one basic variable listed to the side and one nonbasic variable listed above. These numbers are the coefficients in the equations below the tableau defining the basic variables in terms of the nonbasic variables. By multiplying each number of the body of the tableau by the coefficient in its row just to the left of the basic variable, adding products, and subtracting the coefficient at the top of the column, the number at the bottom of the column is obtained. These are all checked, and if they are all negative the optimum has been found. If any are positive, the nonbasic variable at the top of the most positive value is moved over to be a basic variable in the next tableau. One of the basic variables is moved into the vacated place, and these two variables determine what is called the "pivot" in the initial tableau (noted by an asterisk). The pivot is located first by the column that has the largest positive number at the bottom. Secondly, that row is selected where the row constant (to the right of the basic variable) has the smallest ratio to the row element in the specified column. Basic variables with a negative row element are not considered, since making one of these nonbasic would specify a point outside of the set. By this selection method, succeeding tableaux are obtained which are further and further in the direction desired.

The whole process is continued until a tableau is obtained in which all of the numbers at the bottoms of the columns under the nonbasic variables are negative. The procedure outlined has then led to the same optimum that was obtained from the graph. The value at the foot of the column just to the right of the basic variables is obtained by summing the products of the figures in the column with the corresponding figures to the left of the basic variables. This value is the total system weight for the conditions specified. This number is positive even in the final tableau. For a derivation and explanation of the process plus a discussion of certain degenerate cases, see Ref. 2. The weight is 8.9 lb, which agrees with the graphical solution shown in Fig. 1 within the accuracy of these calculations. Since the nonbasic variables in the final tableau are x_2 and x_5, this means that processes represented by these variables, H_2O_2 and KO_2, are zero.

References

[1] Charnes, A., Cooper, W. W., and Mellon, B., "Blending aviation gasolines—a study in programming interdependent activities in an integrated oil company," Econometrica **20**, 135 (April 1952).

[2] Vajda, S., *The Theory of Games and Linear Programming* (John Wiley and Sons Inc., New York, 1956).

Flutter Analysis Using Influence Matrices and Steady-State Aerodynamics

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MATRIX methods long used in solving vibration problems and in solving static aeroelastic problems can be combined to give solutions to the flutter problem. Steady-state aerodynamics is used, with $C_{L\alpha}$ corrected by the magnitude of the Theodorsen lag function. The resulting procedure replaces the typical section approach with one using an influence coefficient description of the deflected surface and finite span aerodynamics. The flutter point is that at which two real eigenvalues merge and continue on as complex conjugates. Earlier work using steady-state aerodynamics and the mode merge criterion is discussed in Refs. 1 and 2.

The vibration problem can be solved by setting up a flexibility influence matrix, say by energy methods, converting to a dynamic matrix and iterating. The square flexibility matrix

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